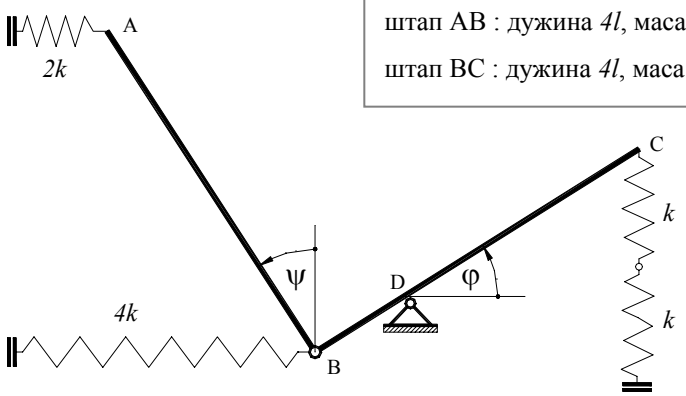
ГРАЂЕВИНСКИ ФАКУЛТЕТ УНИВЕРЗИТЕТА У БЕОГРАДУ
Катедра за Техничку механику и теорију конструкција
Испит из **ТЕХНИЧКЕ МЕХАНИКЕ - 2** **28.01.2008.**

РЕШЕЊЕ ГРУПА **А**

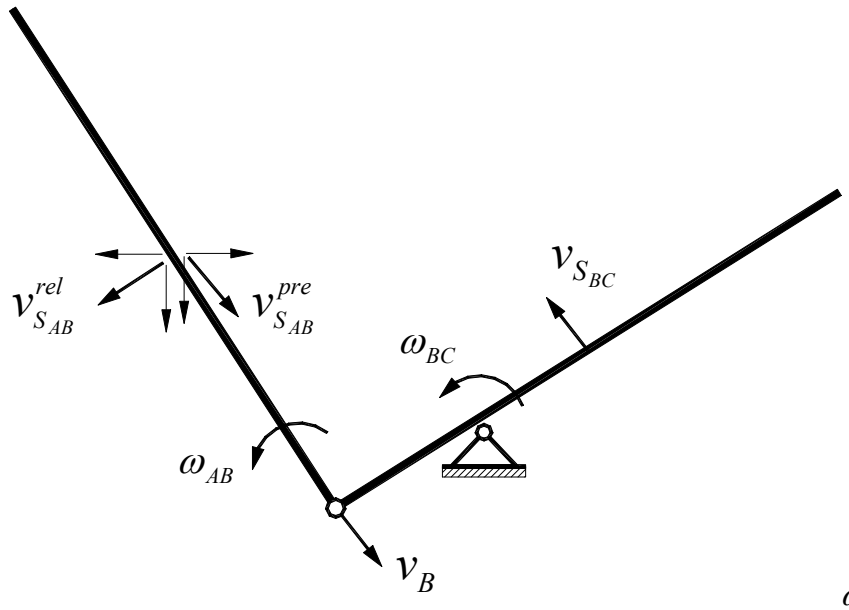
1. ЗАДАТАК: (30%)

штап AB : дужина *4l*, маса *2m*
штап BC : дужина *4l*, маса *2m* (BD = *l*, DC = *3l*)



n = 2
$q_1 = \varphi$
$q_2 = \psi$

* брзине средишта маса и угаоне брзине свих тела



$$\omega_{BC} = \dot{\varphi} \quad \Rightarrow \quad v_{S_{BC}} = \dot{\varphi} \cdot l$$

$$\Rightarrow \quad v_B = \dot{\varphi} \cdot l$$

$$v_{S_{AB}}^{pre} = v_B = \dot{\varphi} \cdot l$$

$$\omega_{AC} = \dot{\psi} \quad \Rightarrow \quad v_{S_{AB}}^{rel} = \dot{\psi} \cdot 2l$$

$$\Rightarrow \quad v_{S_{AB}} = \left(\dot{\varphi} \cdot l \sin\varphi - \dot{\psi} \cdot 2l \cos\psi\right) \cdot \vec{i} + \left(-\dot{\varphi} \cdot l \cos\varphi - \dot{\psi} \cdot 2l \sin\psi\right) \cdot \vec{j}$$

* кинетичка енергија система

$$T_{BC} = \frac{1}{2} \cdot 2m \cdot \left(\dot{\varphi} \cdot l\right)^2 + \frac{1}{2} \cdot \left(\frac{1}{12} \cdot 2m \cdot \left(4l\right)^2\right) \cdot \dot{\varphi}^2 = \frac{7}{3} ml^2 \dot{\varphi}^2$$

$$T_{AB} = \frac{1}{2} \cdot 2m \cdot \left[\left(\dot{\varphi} \cdot l \sin\varphi - \dot{\psi} \cdot 2l \cos\psi\right)^2 + \left(-\dot{\varphi} \cdot l \cos\varphi - \dot{\psi} \cdot 2l \sin\psi\right)^2\right] + \frac{1}{2} \cdot \left(\frac{1}{12} \cdot 2m \cdot \left(4l\right)^2\right) \cdot \left(\dot{\psi}\right)^2 =$$

$$= \ldots = ml^2 \dot{\varphi}^2 + \frac{16}{3} ml^2 \dot{\psi}^2 + 4ml^2 \dot{\varphi}\dot{\psi} \cdot \left(\cos\varphi \sin\psi - \sin\varphi \cos\psi\right)$$

$$T = \sum T_i = \frac{10}{3} ml^2 \dot{\varphi}^2 + \frac{16}{3} ml^2 \dot{\psi}^2 + 4ml^2 \dot{\varphi}\dot{\psi} \cdot \left(\cos\varphi \sin\psi - \sin\varphi \cos\psi\right)$$

силе у опругама

- редно везане опруге: $\Delta_1 = 3l \cdot \sin\varphi$

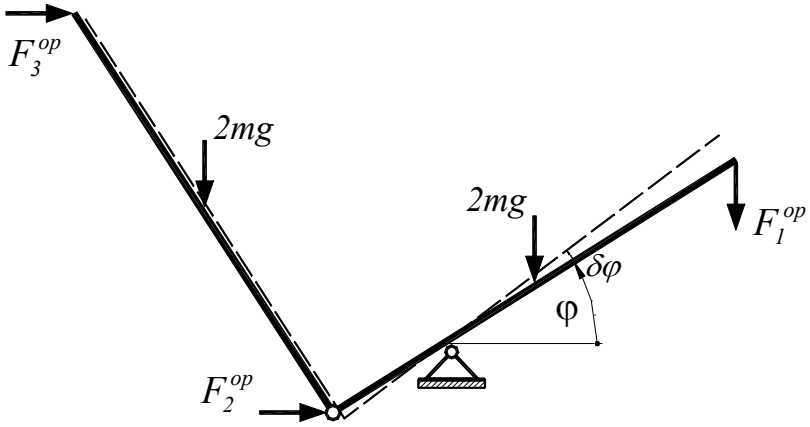
$$\frac{1}{k_1} = \frac{1}{k} + \frac{1}{k} \quad \Rightarrow \quad k_1 = \frac{1}{2}k \qquad\qquad \Rightarrow \quad F_1^{op} = \frac{1}{2}k \cdot 3l\sin\varphi$$

- опруга 2: $\Delta_2 = l - l\cos\varphi \quad \Rightarrow \quad F_2^{op} = 4k \cdot (l - l\cos\varphi)$

- опруга 3: $\Delta_3 = l - l\cos\varphi + 4l\sin\psi \quad \Rightarrow \quad F_3^{op} = 2k \cdot (l - l\cos\varphi + 4l\sin\psi)$

* генералисане силе

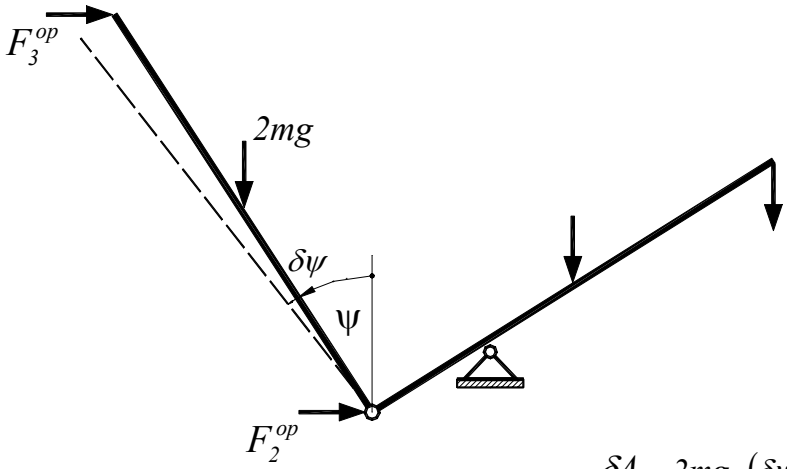
$Q_\varphi = ? \quad (\delta\varphi \neq 0, \delta\psi = 0)$



$$\delta A = -2mg \cdot (\delta\varphi \cdot l\cos\varphi) - F_1^{op} \cdot (\delta\varphi \cdot 3l\cos\varphi) + F_2^{op} \cdot (\delta\varphi \cdot l\sin\varphi) +$$
$$+ 2mg \cdot (\delta\varphi \cdot l\cos\varphi) + F_3^{op} \cdot (\delta\varphi \cdot l\sin\varphi) = Q_\varphi \delta\varphi$$

$$\Rightarrow \quad \ldots \quad \Rightarrow Q_\varphi = kl^2\left(6\sin\varphi - \frac{21}{2}\sin\varphi\cos\varphi + 8\sin\varphi\sin\psi\right)$$

$Q_\psi = ? \quad (\delta\psi \neq 0, \delta\varphi = 0)$



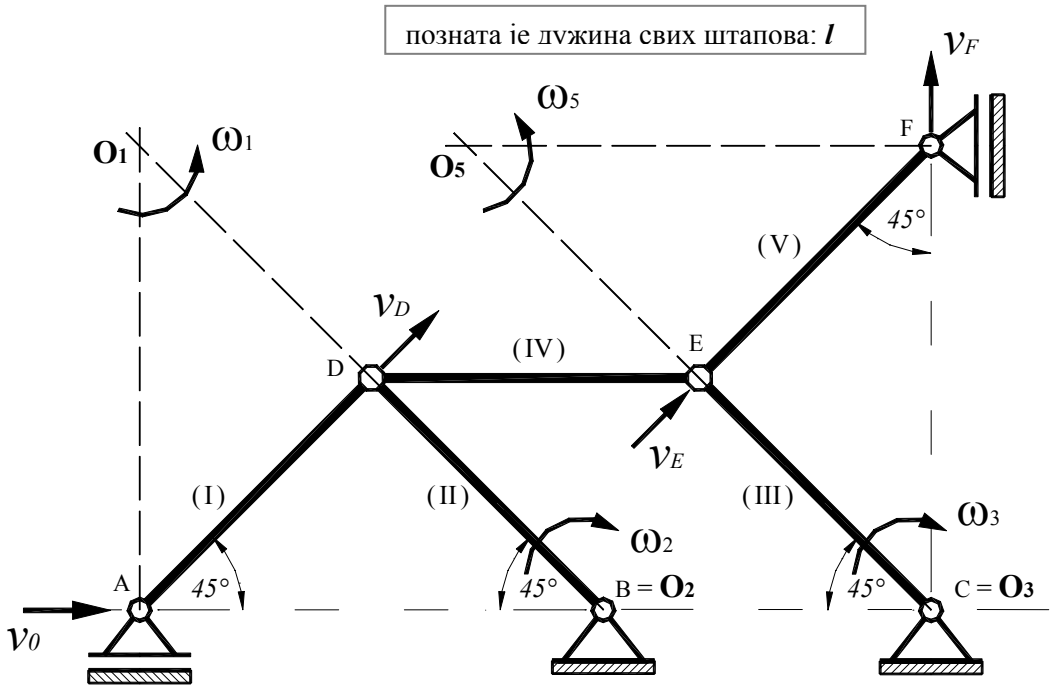
$$\delta A = 2mg \cdot (\delta\psi \cdot 2l\sin\psi) - F_3^{op} \cdot (\delta\psi \cdot 4l\cos\psi) = Q_\psi \delta\psi$$

$$\Rightarrow \quad Q_\psi = 2mgl\sin\psi + kl^2(-8\cos\psi + 8\cos\varphi\cos\psi - 16\sin\psi\cos\psi)$$

4. ЗАДАТАК: (30%)

* <u>број степени слободе кретања система</u>: **n=1**

* <u>брзине и угаоне брзине</u>



$$v_A = \omega_1 \cdot \overline{O_1 A} = \omega_1 \cdot l\sqrt{2} \quad \Rightarrow \quad \omega_1 = \frac{\sqrt{2}}{2}\frac{v_0}{l}$$

$$v_D = \omega_1 \cdot \overline{O_1 D} = \quad \frac{\sqrt{2}}{2}\frac{v_0}{l} \cdot l = \quad \frac{\sqrt{2}}{2} v_0$$

$$v_D = \omega_2 \cdot \overline{O_2 D} = \omega_2 \cdot l \quad \Rightarrow \quad \omega_2 = \frac{\sqrt{2}}{2}\frac{v_0}{l}$$

$$O_4 \to \infty \quad \Rightarrow \quad \omega_4 = 0$$

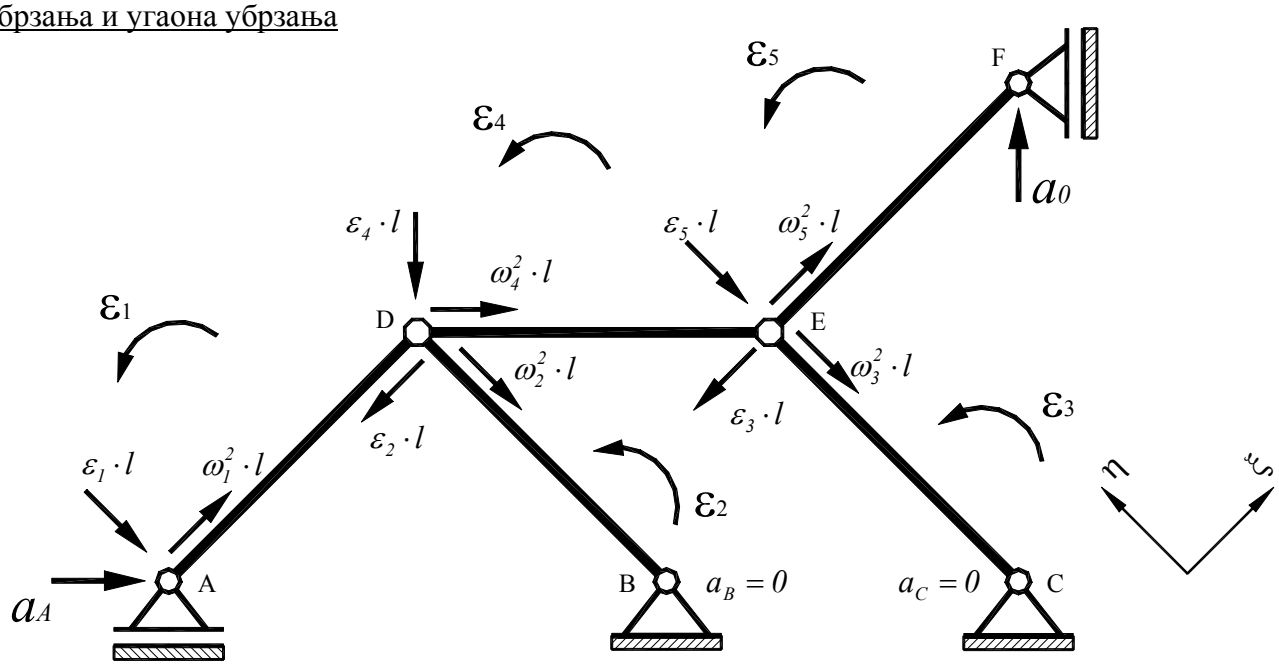
$$\omega_4 = 0 \quad \Rightarrow \quad v_E = v_D = \frac{\sqrt{2}}{2} v_0$$

$$v_E = \omega_3 \cdot \overline{O_3 E} = \omega_3 \cdot l \quad \Rightarrow \quad \omega_3 = \frac{\sqrt{2}}{2}\frac{v_0}{l}$$

$$v_E = \omega_3 \cdot \overline{O_5 E} = \omega_5 \cdot l \quad \Rightarrow \quad \omega_5 = \frac{\sqrt{2}}{2}\frac{v_0}{l}$$

$$v_F = \omega_5 \cdot \overline{O_5 F} = \quad \frac{\sqrt{2}}{2}\frac{v_0}{l} \cdot l\sqrt{2} \quad \Rightarrow \quad v_F = v_0$$

* <u>убрзања и угаона убрзања</u>



$$\vec{a}_E = \vec{a}_F + \vec{a}^F_{E,N} + \vec{a}^F_{E,T} \quad (*)$$

$$\vec{a}_E = \vec{a}_C + \vec{a}^C_{E,N} + \vec{a}^C_{E,T} \quad \Rightarrow \quad \vec{a}^F_E = \vec{a}^C_E \quad / \cdot \vec{\lambda}, \vec{\mu}$$

$$\xi: \ a_0 \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2} \frac{v_0}{l} \right)^2 l = -\varepsilon_3 \cdot l \quad \Rightarrow \quad \varepsilon_3 = -\frac{\sqrt{2}}{2} \frac{a_0}{l} - \frac{1}{2} \frac{v_0^2}{l^2}$$

$$\eta: \ a_0 \frac{\sqrt{2}}{2} + -\varepsilon_5 \cdot l = -\left(\frac{\sqrt{2}}{2} \frac{v_0}{l} \right)^2 l \quad \Rightarrow \quad \varepsilon_5 = \frac{\sqrt{2}}{2} \frac{a_0}{l} + \frac{1}{2} \frac{v_0^2}{l^2}$$

$$(*) \quad \Rightarrow \quad a_{E,X} = \frac{\sqrt{2}}{2} \frac{v_0^2}{l} + \frac{1}{2} a_0 \,, \qquad a_{E,Y} = \frac{1}{2} a_0$$

$$\vec{a}_D = \vec{a}_E + \vec{a}^E_{D,N} + \vec{a}^E_{D,T} \quad (**)$$

$$\vec{a}_D = \vec{a}_B + \vec{a}^B_{D,N} + \vec{a}^B_{D,T} \quad \Rightarrow \quad \vec{a}^B_D = \vec{a}^B_D \quad / \cdot \vec{i}, \vec{j}$$

$$x: \ \left(\frac{\sqrt{2}}{2} \frac{v_0^2}{l} + \frac{1}{2} a_0 \right) = \left(\frac{\sqrt{2}}{2} \frac{v_0}{l} \right)^2 l \cdot \frac{\sqrt{2}}{2} - \varepsilon_2 \cdot l \cdot \frac{\sqrt{2}}{2} \quad \Rightarrow \quad \varepsilon_2 = -\frac{\sqrt{2}}{2} \frac{a_0}{l} - \frac{1}{2} \frac{v_0^2}{l^2}$$

$$y: \ \frac{1}{2} a_0 - \varepsilon_4 \cdot l = -\left(\frac{\sqrt{2}}{2} \frac{v_0}{l} \right)^2 l \cdot \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} \frac{a_0}{l} - \frac{1}{2} \frac{v_0^2}{l^2} \right) \cdot l \cdot \frac{\sqrt{2}}{2} \quad \Rightarrow \quad \varepsilon_4 = 0$$

$$(**) \quad \Rightarrow \quad \vec{a}_D = \vec{a}_E$$

$$\vec{a}_A = \vec{a}_D + \vec{a}^D_{A,N} + \vec{a}^D_{A,T} \quad / \cdot \vec{i}, \vec{j}$$

$$y: \ 0 = \frac{1}{2} a_0 - \varepsilon_1 \cdot l \cdot \frac{\sqrt{2}}{2} - 0 \left(\frac{\sqrt{2}}{2} \frac{v_0}{l} \right)^2 l \cdot \frac{\sqrt{2}}{2} \quad \Rightarrow \quad \varepsilon_1 = \frac{\sqrt{2}}{2} \frac{a_0}{l} + \frac{1}{2} \frac{v_0^2}{l^2}$$

$$x: \left(\frac{\sqrt{2}}{2} \frac{v_0^2}{l} + \frac{1}{2} a_0 \right) = a_A + \left(\frac{\sqrt{2}}{2} \frac{v_0}{l} \right)^2 l \cdot \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2} \frac{a_0}{l} + \frac{1}{2} \frac{v_0^2}{l^2} \right) \cdot l \cdot \frac{\sqrt{2}}{2} \quad \Rightarrow \quad a_A = a_0$$